

I.D. #

Name

Philip Phillips

Subject

Phys. 560

Course

Section

Instructor

Date

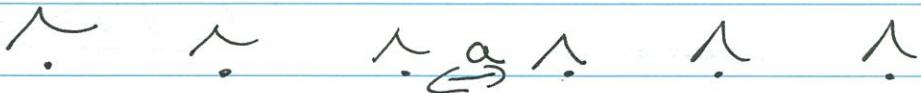
Receiving or giving aid in a final examination is a cause for dismissal from the University.

(Begin Writing on this page)

Lecture 20:

1.) Mott transition: Redux

In the first incarnation, the Mott transition can be envisioned as a lattice of 1D hydrogenic atoms.



When a is small enough a metallic state ensues, else the array is insulating. The criterion for this is $\frac{Ne^2}{\hbar^3} a^3 > 0.38$. This corresponds roughly to a density $0(10^{18} \text{ cm}^{-3})$.

There is another way of thinking about the Mott transition. Consider the array above but with an energy cost for putting two electrons on the same atom.

U can be thought of as

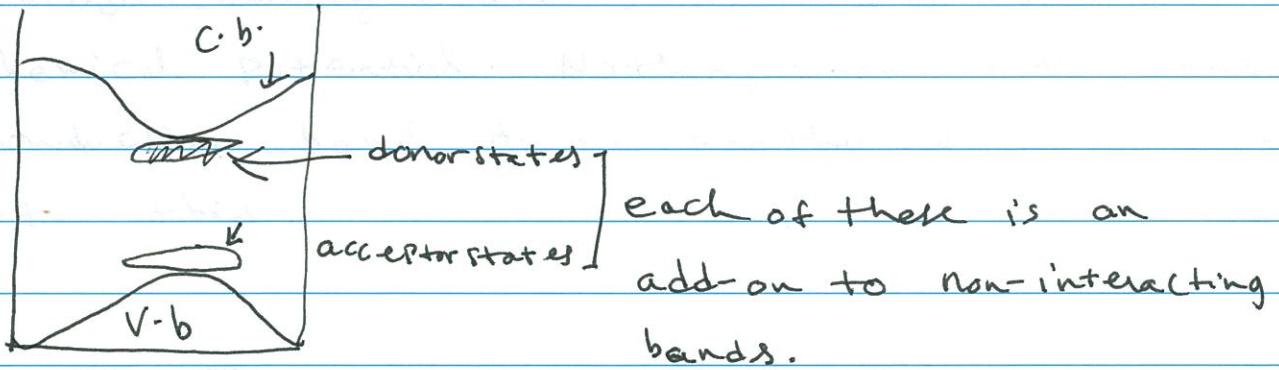
$$U = E^{N+1} + E^{N-1} - 2E^N \gg 0!$$

Mott insulation obtains when U dominates the physics.

The basic model used to understand this kind of Mott transition is the Hubbard model:

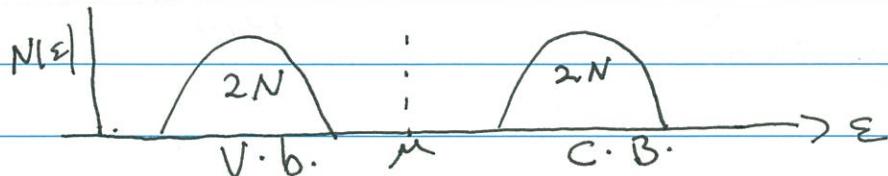
$$H_{\text{Hubb}} = -t \sum_{\langle i,j \rangle, \sigma} |i\rangle\langle j| + \text{h.c.} + U \sum_i n_{i\uparrow} n_{i\downarrow}.$$

Note the difference with the dopant atom model.

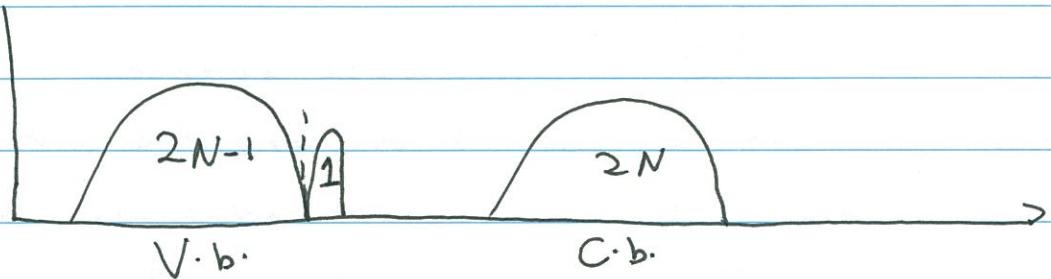


There are no non-interacting bands in the Hubbard model. If an insulating gap forms, then the bands are not eigenstates of a single-particle Hamiltonian. Not so in a Mott system.

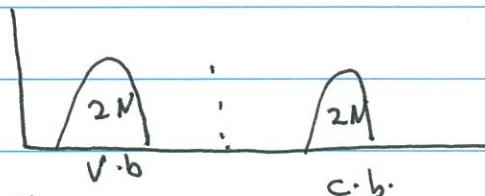
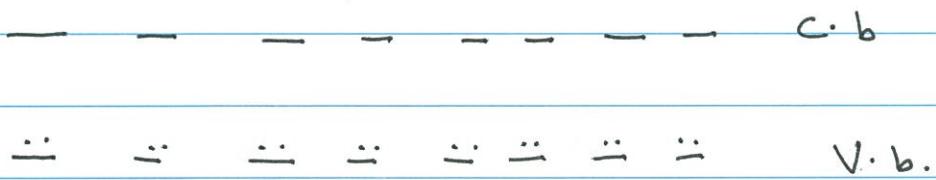
This can be illustrated as follows. In a semiconductor, the states are all non-interacting. Let's say there are N sites. \Rightarrow there are $2N$ ψ 's that can occupy the V.b. and the C.b.



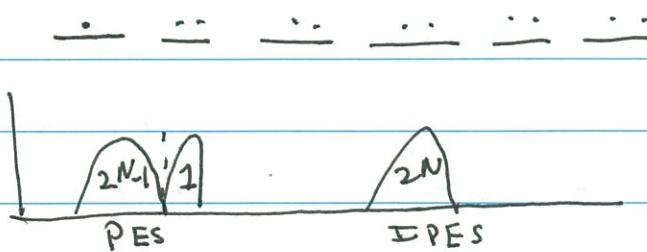
Now consider removing an electron from the V.b.
The new d.o.s. looks like



A single empty state is now pushed above the chemical potential. Nothing changes about the conduction band. Here's another way of looking at this



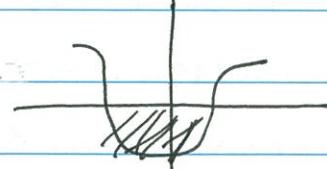
Now let's remove an electron.



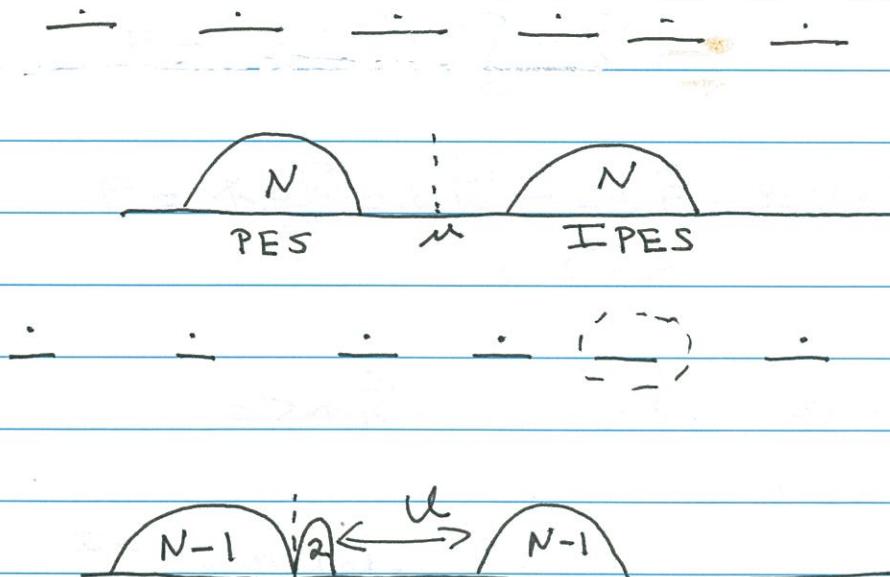
PES = Photo-emission spectrum
removing an electron

IPES = inverse photo-emission
adding an electron.

Now let's do this for a Mott system.

 Let's do a counting argument  $\frac{1}{2}$ -filled band.

assuming the electrons are static: that is no hopping. With 1 electron per site there are N ways of removing an electron from the system. If we were to add an electron, it would cost an energy U .
 \Rightarrow the addition spectrum lies an energy scale U above the removal spectrum. Let's now compare this to the situation when we remove an electron



\Rightarrow removing an electron from the PES part of the spectrum does not leave the IPES unchanged. The two empty states lying above the chemical potential correspond to the two ways

of occupying the empty state with a \uparrow or \downarrow electron. One of these states comes from the PES while the other from the IPES. This does not happen in a semicond. This illustrates that the bands in a Mott system are not single-particle like. They represent composite excitations which do not arise from diagonalizing a S-P Hamiltonian. This is Mottness.

2.) Magnetic Field

a.) semiclassical motion

$$\mathbf{F} = -e(\vec{\mathbf{E}} + \frac{\vec{\mathbf{V}}}{c} \times \vec{\mathbf{B}})$$

$$\mathbf{F} = m \frac{d\vec{\mathbf{r}}}{dt} = m \dot{\vec{\mathbf{r}}} = -e \vec{\mathbf{E}} - \frac{e}{c} \vec{\mathbf{V}} \times \vec{\mathbf{B}}$$

set $\vec{\mathbf{E}} = 0$

$$\Rightarrow m \ddot{\vec{\mathbf{r}}} = -\frac{e}{c} \vec{\mathbf{V}} \times \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{r}} \perp \rightarrow \vec{\mathbf{V}} \text{ and } \vec{\mathbf{B}}.$$

recall $\vec{\mathbf{J}} = \frac{1}{m} \nabla_{\mathbf{r}} \Sigma_{\mathbf{r}}$

In general,

$$m \ddot{\vec{\mathbf{r}}}_i = -\frac{e}{c} \epsilon_{ijk} V_j B_k$$

$\vec{\mathbf{r}}$ oscillates as a function of time.

$$\frac{dt}{d\mathbf{r}_k} = -\frac{m^2 c}{e} \frac{1}{B(\nabla_{\mathbf{r}} \Sigma)_{\perp}}$$

(6.)

$$\Rightarrow \int_{T_1}^{T_2} dt = -\frac{k^2 c}{eB} \int_{K_1}^{K_2} \frac{dk}{(\nabla E_k)_{\perp}}.$$

$$= -\frac{k^2 c}{eB} \int_{K_1}^{K_2} dk \frac{dK_d K_{\perp}}{dE_k}$$

$$\Rightarrow T = -\frac{k^2 c}{eB} \frac{\Delta A}{\Delta E} \quad \Delta A = \pi k^2$$

$$\Delta E = (k^2)^2 / 2m.$$

$$= -\frac{k^2 c}{eB} \frac{\pi k^2}{(k^2)^2} \frac{2m}{(k^2)^2}$$

$$= -\frac{2mc\pi}{eB}.$$

Let $\omega_c = 2\pi/T$ this is the cyclotron frequency.

b.) de-Haas-van Alphen Effect

$$k \dot{r} = -\frac{e}{c} \vec{J} \times \vec{B}$$

$$\text{Let } \vec{J} = \vec{r}.$$

$$\Rightarrow k \dot{r} = -\frac{e}{c} \dot{r} \times B$$

$$\Rightarrow k(r(t) - r(0)) = -\frac{e}{c} (r(t) - r(0)) \times B$$

$$\Rightarrow B \times (r(t) - r(0)) = -\frac{e}{kc} B \times [r(t) - r(0)] \times B.$$

$$\text{recall } A \times (B \times C) = B(A \cdot C) - C(A \cdot B).$$

$$\Rightarrow B \times [k_1 t - k_{10}] = \frac{-e}{hc} (r_1 t - r_{10}) B^2 + \frac{e B \cdot (r_1 t - r_{10}) B}{hc}$$

what kinds of orbits are described here?

Bohr-Sommerfeld quantization.

$$\oint p \cdot dr = (n + \gamma) 2\pi \lambda = \frac{e}{c} \Phi_n \quad \text{for electrons.}$$

Φ_n is the magnetic flux $= \frac{2\pi \lambda c}{e} (n + \gamma)$.

Let's prove this.

$$\vec{p} = \lambda \vec{k} - \frac{e \vec{A}}{c}$$

$$\Rightarrow \lambda \frac{dK}{dt} = -\frac{e}{c} \frac{d\vec{r}}{dt} \times \vec{B}$$

$$\Rightarrow \lambda K = -\frac{e}{c} \int \vec{r} \times \vec{B} \cdot d\vec{r} + C$$

$$\int \lambda K \cdot dr = -\frac{e}{c} \int \vec{r} \times \vec{B} \cdot d\vec{r}$$

$$= \frac{e}{c} B \cdot \int r \times dr$$

$\int r \times dr = 2 \text{ (area enclosed)}$.

$$\Rightarrow \int \lambda K \cdot dr = \frac{2e}{c} \Phi$$

$$\Rightarrow \oint p \cdot dr = \int \lambda K \cdot dr - \frac{e}{c} \int A \cdot dr$$

$$= \frac{2e}{c} \Phi_r - \frac{e}{c} \Phi_n = \frac{e}{c} \Phi_n$$

From $\nabla \times \mathbf{B} = -\frac{e}{c} \frac{d\mathbf{r}}{dt} \times \mathbf{B}$, we can relate an increment in \mathbf{K} -space to an increment in \mathbf{r} -space.

$$\Rightarrow \Delta \mathbf{K} = \frac{e}{c} \Delta r \mathbf{B} \Rightarrow \Delta r = \frac{c}{eB} \Delta \mathbf{K}.$$

\Rightarrow the areas in \mathbf{K} and \mathbf{r} -space are related, namely

$$A_n = \left(\frac{kc}{eB} \right)^2 S_n ; \quad S_n \text{ is the } \mathbf{K}\text{-space area.}$$

The area is related to the magnetic flux.

$$\frac{eB}{c} \int \sigma r dr = 2 \frac{eB}{c} A_n = \frac{e}{c} \Phi_n$$

$$\Rightarrow \Phi_n = B A_n.$$

$$\Rightarrow \Phi_n = \left(\frac{kc}{e} \right)^2 \frac{1}{B} S_n.$$

$$\Rightarrow S_n = B \left(\frac{2\pi e}{kc} \right) (n + \gamma).$$

In F.S. experiments, we are interested in values of the field for which the area does not change.

Let

$$\frac{S}{B_n} = \frac{2\pi e}{kc} (n + \gamma)$$

$$\Rightarrow \frac{S}{B_{n+1}} - \frac{S}{B_n} = \frac{2\pi e}{kc} \Rightarrow \text{oscillations with a period of } \frac{1}{B} \text{ when}$$

8

9

The field is incremented by $2\pi e/hc \cdot S$ in
the Fermi surface volume.

$$\frac{S_{n+1} - S_n}{\Delta t} = \frac{2\pi e B}{hc}$$

$$nB^2 \left(\frac{\partial \Phi}{\partial B} \right) = \Delta \Phi$$

$$\frac{\Phi_{n+1} - \Phi_n}{\Delta t} = nB^2 \left(\frac{\partial \Phi}{\partial B} \right)$$

$$\Delta \Phi = \Phi_{n+1} - \Phi_n$$

$$nB^2 \left(\frac{\partial \Phi}{\partial B} \right) = \Delta \Phi$$

$$(n+1) \left(\frac{\partial \Phi}{\partial B} \right) = \Delta \Phi$$

$$\frac{\partial \Phi}{\partial B} = \frac{\Delta \Phi}{n+1}$$

$$\frac{\partial \Phi}{\partial B} = \frac{\Delta \Phi}{n+1}$$

$$\frac{\partial \Phi}{\partial B} = \frac{\Delta \Phi}{n+1}$$

$$\frac{\partial \Phi}{\partial B} = \frac{\Delta \Phi}{n+1} = \frac{2\pi S}{h} = \frac{2\pi S}{h} \cdot \frac{e}{m^2}$$